Working out coordinates using distances

27th January 2018 By An7ar35

Contents

1 Introduction

Using distances between different points we can deduce relative coordinates. Afterwards, if we have an absolute coordinate for one of the points, we can easily convert the rest to their absolute values.

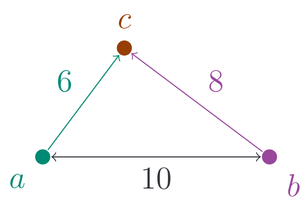
Working out coordinates from distances of points on a plane (x and y) is called ”Trilateration”.

2 Coordinates on a plane (2D)

Our working example problem will be to out where a Wifi hotspot is based on its distance from other known hotspots situated at the same level inside a building.

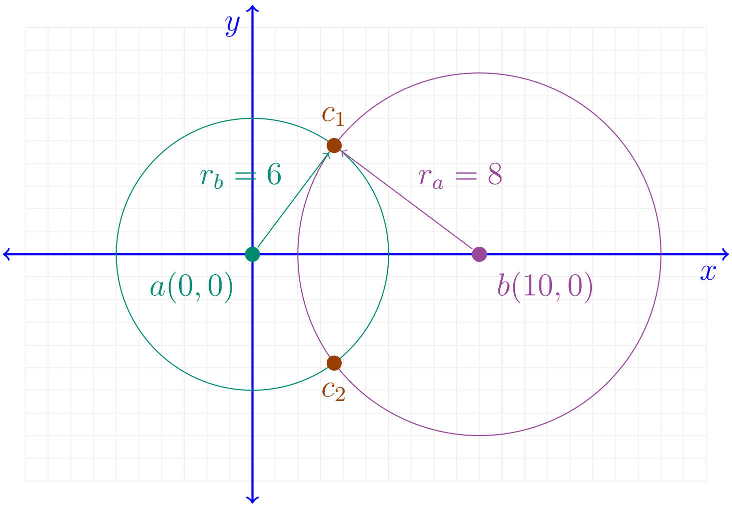
Let’s make the problem a little simpler for the moment by using relative coordinates centred around hotspot a which will be our $(0, 0)$ point in the graph.

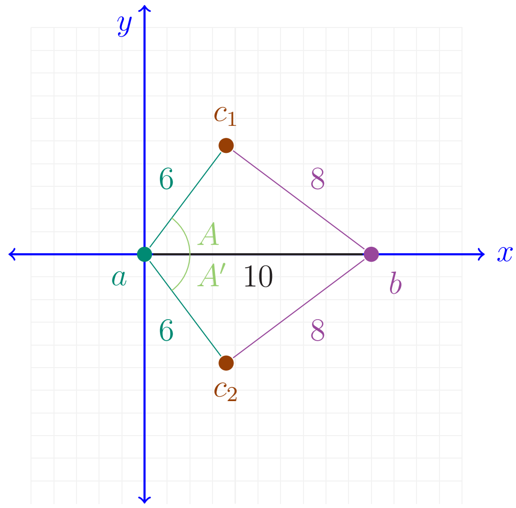
The distance between a and b known to be 10m. The hotspot c, based on signal strength, is calculated to be 6m away from a and 8m away from b.



We can draw circles whose radii equal the distances to point c to see where c might be.

Let’s assign a to coordinates $(0, 0)$ on our graph and place b on the x-axis and draw the circles... There are 2 possible position for c:

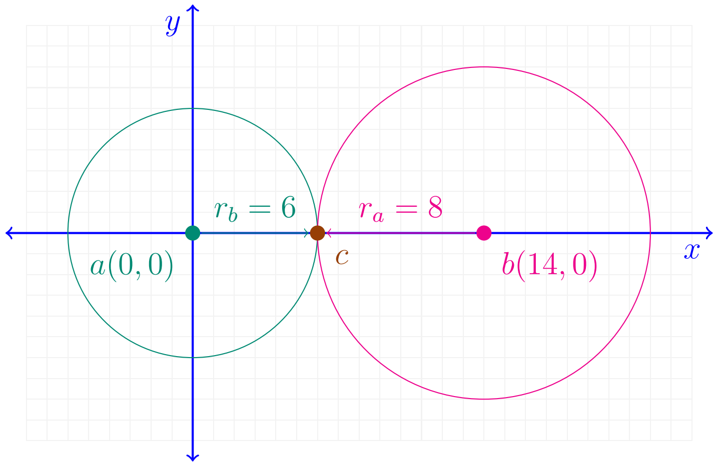




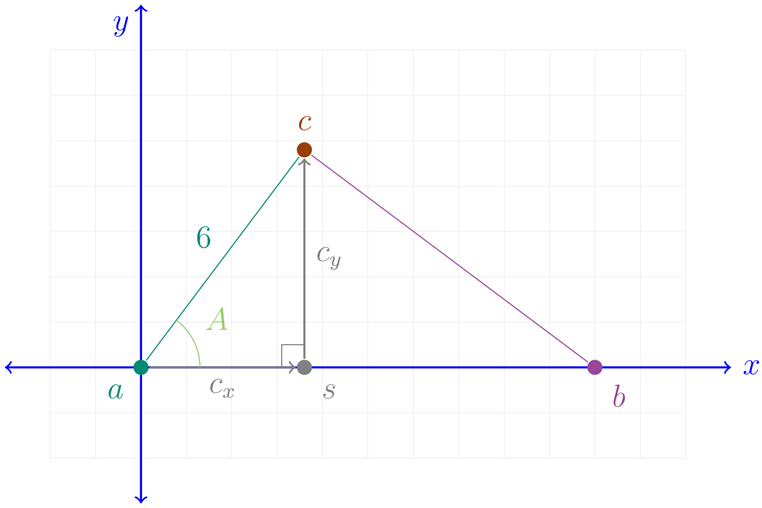
If we have 2 hotspots (a and b), is it enough to calculate a relative coordinate for our unknown hotspot (c)? No, we need another hotspot in order to decide which of c_1 or c_2 is the correct location.

Circle intersection exception

There is one case where we end up with only a single possible location for c: when a and b's distances to c do not overlap.
e.g.: If we shift hotspot b a little further on the x-axis and still detect c 8m away we end up with circles just touching instead of intersecting.



Using a bit of trigonometry (Law of the Cosines), we can calculate angle A.



$$\begin{aligned}\angle A &= \cos^{-1}\left(\frac{ac^2 + ab^2 - bc^2}{2 \times ac \times ab}\right)\\ &= \cos^{-1}\left(\frac{6^2 + 10^2 - 8^2}{2 \times 6 \times 10}\right)\\ &= \cos^{-1}\left(\frac{72}{120}\right)\\ &= \cos^{-1}\left(0.6\right)\\ &= \mathbf{53.1301023542}^\circ\end{aligned}$$

Calculating the (x, y) coordinates of c is then a trivial matter.

$$\begin{aligned}c_x &= ac \times \cos A\\ &= 6 \times 0.6\\ &= \mathbf{3.6}\end{aligned}$$

$$\begin{aligned}c_y &= ac \times \sin A\\ &= 6 \times 0.8\\ &= \pm\mathbf{4.8}\end{aligned}$$

As we can have 2 possible coordinates for c, $(3.6, 4.8)$ and $(3.6, -4.8)$, we need a fourth Wifi hotspot (d) to decide which of c_1 or c_2 is the correct one.

Hotspot d, whose distance to c is measured at 9m, is added to the graph. To check which c coordinate point is correct we can calculate c_1 and c_2's distance to d and see which one matches the 9m.

We can use the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for that.

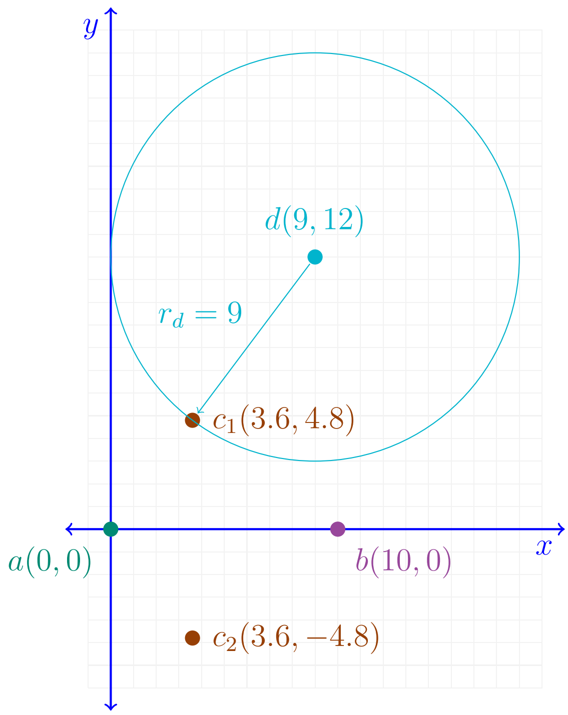
Let's calculate for c_1:

$$c_1 d = \sqrt{(9-3.6)^2 + (12-4.8)^2}$$
$$= \sqrt{29.16 + 51.84}$$
$$= \mathbf{9}$$

And for c_2:

$$c_2 d = \sqrt{(9-3.6)^2 + (12-(-4.8))^2}$$
$$= \sqrt{29.16 + 282.24}$$
$$= \mathbf{17.647}$$

$c_1 d$ matches the known distance of 9m. We can now deduce that c_1 is the correct coordinate for c. If we plot it on the graph this becomes obvious.



We now have the relative coordinates of hotspot c on top of the others located on the same plane $(z = 0)$:

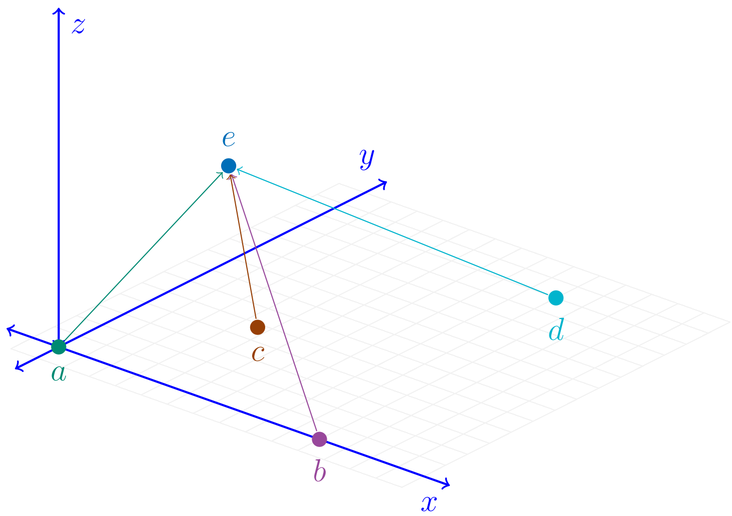
- a = $(0, 0, 0)$
- b = $(0, 10, 0)$
- c = $(3.6, 4.8, 0)$
- d = $(9, 12, 0)$

3 Coordinates space (3D)

What if we add a Wifi hotspot (e) in one of the floors above? This adds another dimension to the problem (ahr ahr, get it? 😊). This can be solved by expanding trilateration calculations to accommodate the 3rd dimension.

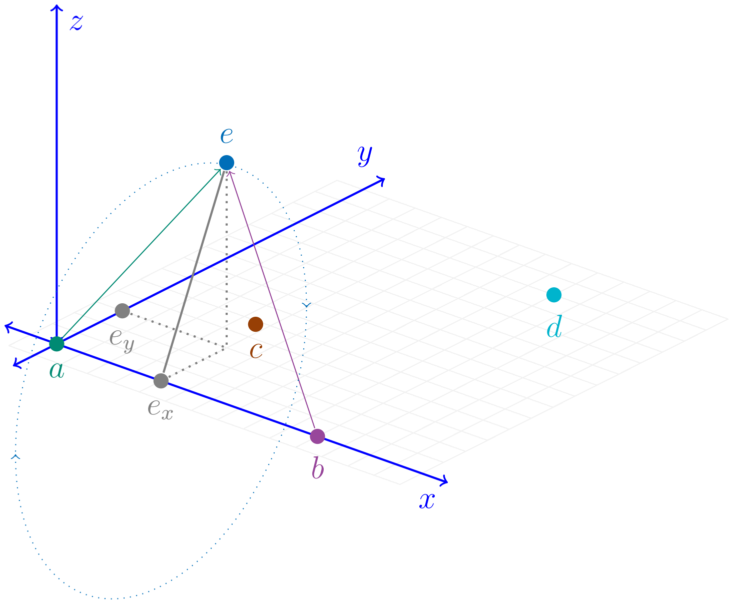
The measurements of our known hotspots to the new one (e) are:

- ae = $7.81025m$
- be = $9m$
- ce = $6.27694m$
- de = $11.91638m$



Calculating the (x, y, z) coordinates of e...

As the base of our triangle is aligned on the x axis, the position of e in the other dimensions (y and z) will not have an effect on where e_x is located.

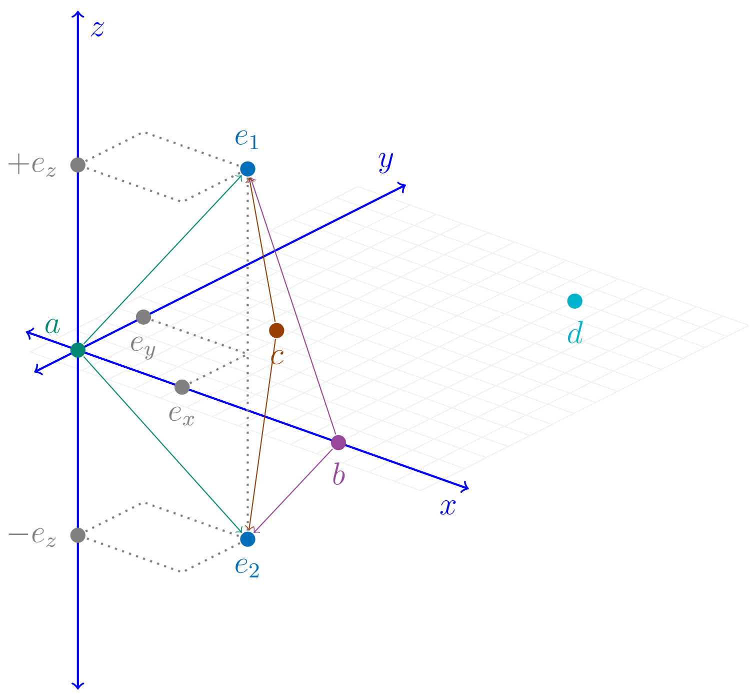


$$\begin{aligned} e_x &= \frac{ae^2 - be^2 + ab^2}{2 \times ab} \\ &= \frac{7.81025^2 - 9^2 + 10^2}{2 \times 10} \\ &= \frac{80}{20} \\ &= \mathbf{4} \end{aligned}$$

Now for e_y:

$$\begin{aligned} e_y &= \frac{ae^2 - ce^2 + {c_y}^2 + {c_x}^2 - \dfrac{c_x \times (ae^2 - be^2 + ab^2)}{ab}}{2 \times c_y} \\ &= \frac{7.81025^2 - 6.27694^2 + 4.8^2 + 3.6^2 - \dfrac{3.6 \times (7.81025^2 - 9^2 + 10^2)}{10}}{2 \times 4.8} \\ &= \frac{57.6 - 28.8}{9.6} \\ &= \mathbf{3} \end{aligned}$$

And finally for e_z:



$$\begin{aligned} e_z &= \sqrt{ae^2 - {e_x}^2 - {e_y}^2} \\ &= \sqrt{7.81025^2 - 4^2 - 3^2} \\ &= \sqrt{36} \\ &= \pm\mathbf{6} \end{aligned}$$

As with point c in the 2-dimensional problem, we end up with 2 possibilities, e_1 and e_2, but this time based on their z positions.

Point d cannot help us as it lies within the same plane as the other points (a, b and c) that form the base of the tetrahedrons[1] (a, b, c, e_1) and (a, b, c, e_2). i.e.: the distances de_1 and de_2 are the same.

We need the help of another point (or "hotspot" in our case) that lie somewhere else on the z axis (i.e.: $z \neq 0$) to help discriminate between e_1 and e_2.

Let's say we have hotspot f in the basement located at $(6, 8, -2)$ and whose distance to e is 9.644m. With this we can work out using distances which, of points e_1 or e_2, is the correct one.

Calculating 3D distance

To calculate distance between two 3D points is similar to the way we do it in 2 dimensions. We just need to add the z dimension values to the equation:

$$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

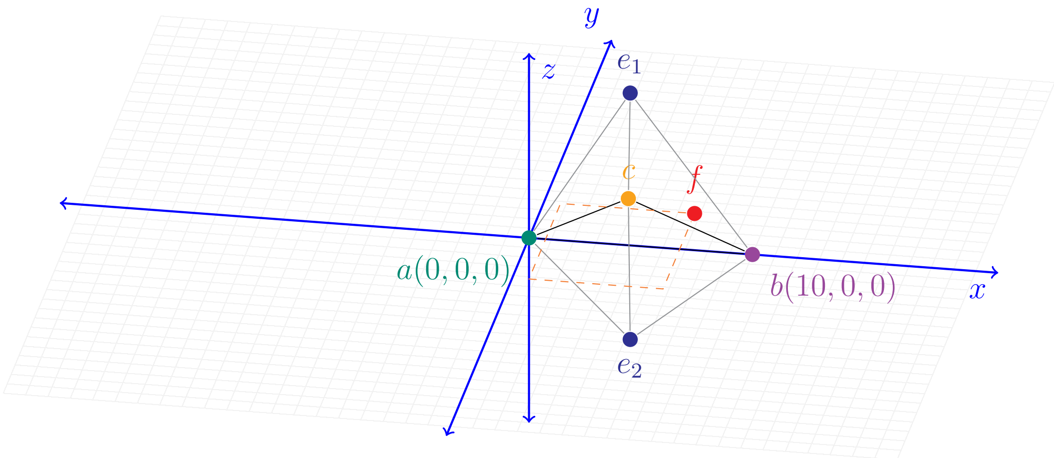
$$\text{distance } e_1 f = \sqrt{(4-6)^2 + (3-8)^2 + (6-(-2))^2}$$

$$= \mathbf{9.644}$$

$$\text{distance } e_2 f = \sqrt{(4-6)^2 + (3-8)^2 + ((-6)-(-2))^2}$$

$$= \mathbf{6.708}$$

Since $e_1 f$ is the same as the recorded distance ef we can deduce that $e_1 = (4, 3, 6)$ is the correct coordinate for e.



[1] **Tetrahedron**: 4 sides, all triangles.

4 Transformation from absolute to relative coordinates

Points with absolute values will most likely not line up on the xy plane of a graph. In order to facilitate trilateration we will need to line things up through a bit of transformation.

4.1 Order and shifting

First we need to decide in which order the points of the base triangle will be set. Points a and b should be on either ends of the hypotenuse (longest vertex) where $a_x < b_x$.

The base triangle abc is moved so that a is at $(0, 0, 0)$.

Adding/Subtracting 3D coordinates

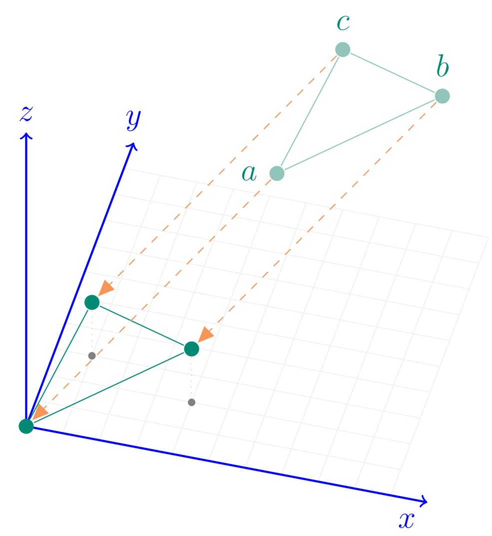
Adding and subtracting coordinates is very easy. Just add/subtract the individual parts of the coordinates. For example:

Shifting coordinate $a(1, 2, 4)$ by $s(5, 4, 3)$:

$$\begin{aligned} b &= (a_x + s_x), (a_y + s_y), (a_z + s_z) \\ &= (1 + 5), (2 + 4), (4 + 3) \\ &= (6, 6, 7) \end{aligned}$$

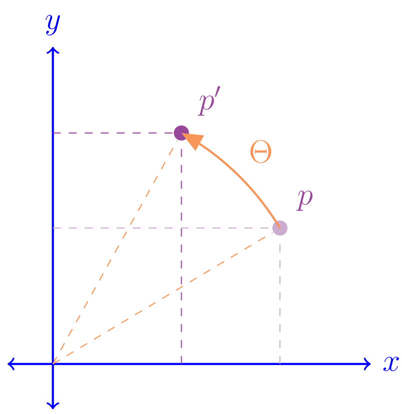
Then to shift back:

$$\begin{aligned} a &= (b_x - s_x), (b_y - s_y), (b_z - s_z) \\ &= (6 - 5), (6 - 4), (7 - 3) \\ &= (1, 2, 4) \end{aligned}$$



4.2 Point Rotation

During rotations we need to calculate the angles. We can use simple right-angle trigonometry (a.k.a. Soh-Cah-Toa) to get those.
Positive rotation is counter-clockwise. To rotate clockwise we need to set the angle of rotation as negative.
E.g.: If we want to rotate a point on an axis by $30°$ clockwise then we need to put a minus sign in front (-30) of it in our rotation equations.

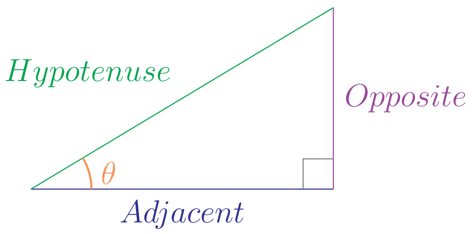


SOH-CAH-TOA

$$\sin \theta = \frac{Opposite}{Hypotenuse}$$

$$\cos \theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan \theta = \frac{Opposite}{Adjacent}$$



4.2.1 Z-Axis

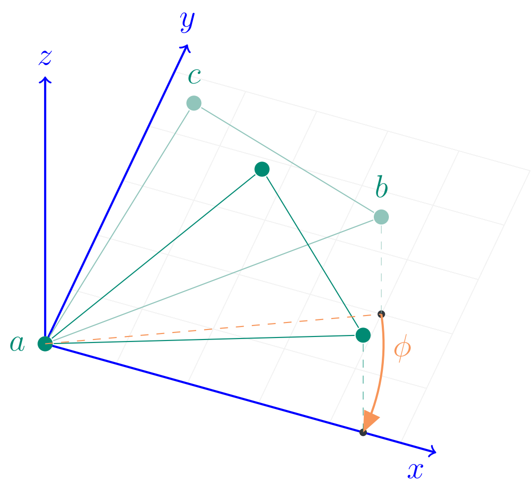
Angle ϕ for the z axis rotation is calculated[2] and abc is rotated putting b at $(ab, 0, b_z)$.

Z-Axis Rotation

$$x' = x \times \cos\left(\frac{-\phi\pi}{180}\right) - y \times \sin\left(\frac{-\phi\pi}{180}\right)$$

$$y' = x \times \sin\left(\frac{-\phi\pi}{180}\right) + y \times \cos\left(\frac{-\phi\pi}{180}\right)$$

$$z' = z$$



4.2.2 Y-Axis

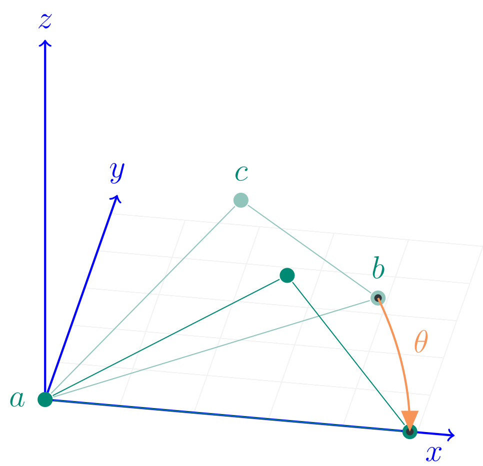
Angle θ for the y axis rotation is calculated and abc is rotated putting b at $(ab, 0, 0)$.

Y-Axis Rotation

$$x' = x \times \cos\left(\frac{-\theta\pi}{180}\right) - z \times \sin\left(\frac{-\theta\pi}{180}\right)$$

$$y' = y$$

$$z' = x \times \sin\left(\frac{-\theta\pi}{180}\right) + z \times \cos\left(\frac{-\theta\pi}{180}\right)$$



[2] We convert angles to radians in the rotation equations with $rad = \frac{\theta\pi}{180}$.

4.2.3 X-Axis

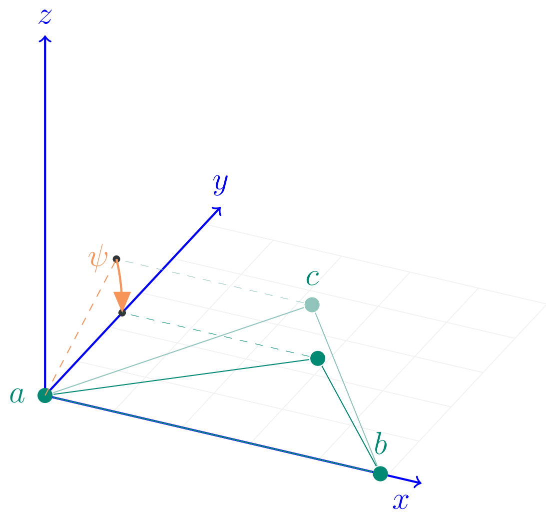
Angle ψ for the x axis rotation is calculated and abc is rotated a third time placing point c's z-coordinate at 0 $(c_x, c_y, 0)$.

X-Axis Rotation

$$x' = x$$

$$y' = y \times \cos\left(\frac{-\psi\pi}{180}\right) - z \times \sin\left(\frac{-\theta\pi}{180}\right)$$

$$z' = y \times \sin\left(\frac{-\psi\pi}{180}\right) + z \times \cos\left(\frac{-\theta\pi}{180}\right)$$



4.3 Reversing

After doing the trilateration to find the e_1 and e_2 coordinates, the steps and calculated transformation values should be applied in reverse (i.e.: x-rotation, y-rotation, z-rotation and shifting with their negative respective values). Working out which one of the two possibilities is again a question of checking with a known distance to a 'helper' point d not situated on the same xy plane as abc.

5 A complete example

5.1 The problem

You are a space explorer/cartographer in the far future. In order to do a hyper-jump you need to first work out the coordinates of your destination.

A new star system has been discovered and you'd prefer getting there quickly with a hyper-jump rather than taking years by using the old fashioned way at sub-light speeds.

Fortunately other explorers have recorded and shared the distance in light years from various outposts to this new system...

Outpost Coordinates

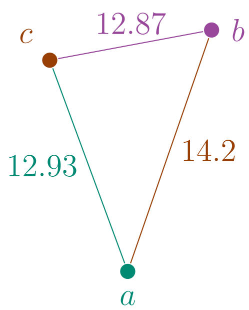
Outpost	x	y	z	Distance
1	-28.75000	25.00000	10.43750	31.1840
2	-24.31250	37.75000	6.03125	20.2887
3	-32.87500	36.15625	15.50000	32.1884
4	-34.09375	26.21875	-5.53125	23.9109

5.2 Distances

Let's use outposts 1, 2 and 3 as our base triangle and 4 as our discriminator. We first need to get the distances between outposts making up the triangle:

$$
\begin{aligned}
1\leftrightarrow 2 &= \sqrt{(-32.875-(-28.75))^2+(36.15625-25)^2+(15.5-10.4375)^2} \\
&= \mathbf{12.92697} \\
1\leftrightarrow 3 &= \sqrt{(-24.3125-(-28.75))^2+(37.75-25)^2+(6.03125-10.4375)^2} \\
&= \mathbf{14.20102} \\
2\leftrightarrow 3 &= \sqrt{(-24.3125-(-32.875))^2+(37.75-36.15625)^2+(6.03125-15.5)^2} \\
&= \mathbf{12.86521}
\end{aligned}
$$

Since the distance between outpost 1 and 2 is the largest (hypotenuse), these will make up our triangle's base.
As $1_x < 2_x$ we will assign a to 1 and b to 2. Outpost 3 therefore becomes c.



Outpost	Assigned letter
1	a
2	b
3	c
4	d

5.3 Transformation

5.3.1 Shifting the points

Next we move our triangle so that a is at $(0, 0, 0)$. Since a is $(-28.75000, 25.00000, 10.43750)$, this will be the amount to shift all the points in our triangle by.

$a =$ **(0,0,0)**

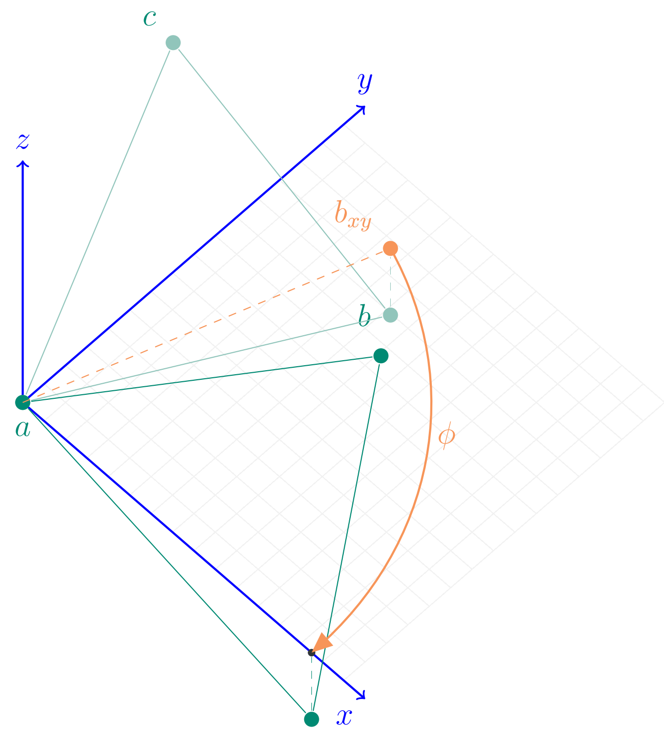
$b = (-24.3125 - (-28.75)), (37.75 - 25), (6.03125 - 10.4375)$

$=$ **(4.4375,12.75,−4.40625)**

$c = (-32.875 - (-28.75)), (36.15625 - 25), (15.5 - 10.4375)$

$=$ **(-4.125,11.15625,5.0625)**

5.3.2 Z-Axis Rotation



Angle ϕ is calculated:

$$\phi = \tan^{-1} \frac{b_y}{b_x}$$

$$= \tan^{-1} \frac{12.75}{4.4375}$$

$$= \mathbf{70.81}^\circ$$

Then the new rotated coordinates for b and c:

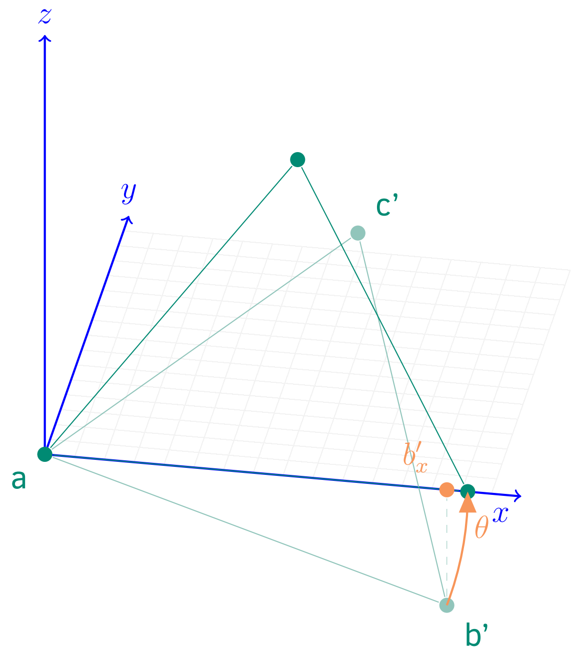
$$b'_x = 4.4375 \times \cos\left(\frac{-70.81 \times \pi}{180}\right) - 12.75 \times \sin\left(\frac{-70.81 \times \pi}{180}\right) = \mathbf{13.500145}$$

$$b'_y = 4.4375 \times \sin\left(\frac{-70.81 \times \pi}{180}\right) + 12.75 \times \cos\left(\frac{-70.81 \times \pi}{180}\right) = \mathbf{0}$$

$$c'_x = -4.125 \times \cos\left(\frac{-70.81 \times \pi}{180}\right) - 11.15625 \times \sin\left(\frac{-70.81 \times \pi}{180}\right) = \mathbf{9.1804441}$$

$$c'_y = -4.125 \times \sin\left(\frac{-70.81 \times \pi}{180}\right) + 11.15625 \times \cos\left(\frac{-70.81 \times \pi}{180}\right) = \mathbf{7.5628689}$$

5.3.3 Y-Axis Rotation



Angle θ is calculated:

$$\theta = \tan^{-1}\frac{b'_z}{b'_x} = \tan^{-1}\frac{-4.40625}{13.50015} = \textbf{-18.07592}^\circ$$

Then the new rotated coordinates for b and c:

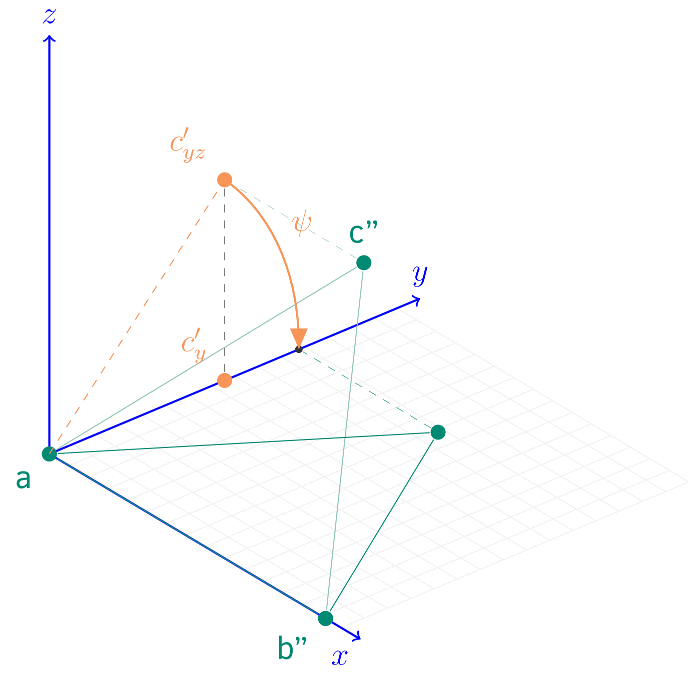
$$b''_x = 13.50015 \times \cos\left(\frac{18.07592 \times \pi}{180}\right) - (-4.40625) \times \sin\left(\frac{18.07592 \times \pi}{180}\right) = \textbf{14.201024}$$

$$b''_z = 13.50015 \times \sin\left(\frac{18.07592 \times \pi}{180}\right) + (-4.40625) \times \cos\left(\frac{18.07592 \times \pi}{180}\right) = \textbf{0}$$

$$c''_x = 9.18044 \times \cos\left(\frac{18.07592 \times \pi}{180}\right) - 5.0625 \times \sin\left(\frac{18.07592 \times \pi}{180}\right) = \textbf{7.1565736}$$

$$c''_z = 9.18044 \times \sin\left(\frac{18.07592 \times \pi}{180}\right) + 5.0625 \times \cos\left(\frac{18.07592 \times \pi}{180}\right) = \textbf{7.6611252}$$

5.3.4 X-Axis Rotation



Angle ψ is calculated:

$$\psi = \tan^{-1} \frac{c''_z}{c''_y}$$
$$= \tan^{-1} \frac{7.6611252}{7.5628689}$$
$$= \mathbf{45.36978}^\circ$$

Then the new rotated coordinates for c:

$$c'''_y = 7.5628689 \times \cos\left(\frac{-45.36978 \times \pi}{180}\right) - 7.6611252 \times \sin\left(\frac{-45.36978 \times \pi}{180}\right) = \mathbf{10.765214}$$

$$c'''_z = 7.5628689 \times \sin\left(\frac{-45.36978 \times \pi}{180}\right) + 7.6611252 \times \cos\left(\frac{-45.36978 \times \pi}{180}\right) = \mathbf{0}$$

5.4 Trilateration

Now we have our base triangle positioned:

- $a = (0, 0, 0)$
- $b = (14.201024, 0, 0)$
- $c = (7.1565736, 10.765214, 0)$

- $ab = 14.20102$
- $ac = 12.92697$
- $bc = 12.86521$

- $ae = 31.1840$
- $be = 20.2887$
- $ce = 32.1884$
- $de = 23.9109$

$$e_x = \frac{ae^2 - be^2 + ab^2}{2 \times ab}$$
$$= \frac{31.184^2 - 20.2887^2 + 14.20102^2}{2 \times 14.20102}$$
$$= \mathbf{26.845941}$$

$$e_y = \frac{ae^2 - ce^2 + {c_y}^2 + {c_x}^2 - \dfrac{c_x \times (ae^2 - be^2 + ab^2)}{ab}}{2 \times c_y}$$

$$= \frac{31.184^2 - 32.1884^2 + 10.765214^2 + 7.1565736^2 - \dfrac{7.1565736 \times (31.184^2 - 20.2887^2 + 14.20102^2)}{14.20102}}{2 \times 10.765214}$$

$$= \frac{103.45514 - 384.2499}{21.530428}$$

$$= \mathbf{-13.041764}$$

$$e_z = \sqrt{ae^2 - {e_x}^2 - {e_y}^2}$$

$$= \sqrt{31.184^2 - 26.845941^2 - (-13.041764)^2}$$

$$= \sqrt{81.6497}$$

$$= \pm\mathbf{9.0360223}$$

5.5 Transforming back to absolute coordinates

Now we need to transform in reverse our coordinates for e using the angles and the shifting values.

e_1''': $(26.845941, -13.041764, 9.0360223)$

e_2''': $(26.845941, -13.041764, -9.0360223)$

x axis ψ: $45.36978°$

y axis θ: $-18.07592°$

z axis ϕ: $70.81°$

position shift: $(-28.75, 25, 10.4375)$

5.5.1 X-Axis Rotation

$$e''_x = e'''_x$$
$$= \mathbf{26.845941}$$

$$e''_{y1} = e'''_y \times \cos\left(\frac{\psi\pi}{180}\right) - e'''_{z1} \times \sin\left(\frac{\psi\pi}{180}\right)$$
$$= -13.041764 \times \cos\left(\frac{45.36978\pi}{180}\right) - 9.0360223 \times \sin\left(\frac{45.36978\pi}{180}\right)$$
$$= \mathbf{-15.592839}$$

$$e''_{y2} = e'''_y \times \cos\left(\frac{\psi\pi}{180}\right) - e'''_{z2} \times \sin\left(\frac{\psi\pi}{180}\right)$$
$$= -13.041764 \times \cos\left(\frac{45.36978\pi}{180}\right) - (-9.0360223) \times \sin\left(\frac{45.36978\pi}{180}\right)$$
$$= \mathbf{-2.7321666}$$

$$e''_{z1} = e'''_y \times \sin\left(\frac{\psi\pi}{180}\right) + e'''_{z1} \times \cos\left(\frac{\psi\pi}{180}\right)$$
$$= -13.041764 \times \sin\left(\frac{45.36978\pi}{180}\right) + 9.0360223 \times \cos\left(\frac{45.36978\pi}{180}\right)$$
$$= \mathbf{-2.9326908}$$

$$e''_{z2} = e'''_y \times \sin\left(\frac{\psi\pi}{180}\right) + e'''_{z2} \times \cos\left(\frac{\psi\pi}{180}\right)$$
$$= -13.041764 \times \sin\left(\frac{45.36978\pi}{180}\right) + (-9.0360223) \times \cos\left(\frac{45.36978\pi}{180}\right)$$
$$= \mathbf{-15.629222}$$

Results

Our two possible e points:

$$e''_1 = (26.845941, -15.592839, -2.9326908)$$
$$e''_2 = (26.845941, -2.7321666, -15.629222)$$

5.5.2 Y-Axis Rotation

$$e'_{x1} = e''_x \times \cos\left(\frac{\theta\pi}{180}\right) - e''_{z1} \times \sin\left(\frac{\theta\pi}{180}\right)$$

$$= 26.845941 \times \cos\left(\frac{-18.07592\pi}{180}\right) - (-2.9326908) \times \sin\left(\frac{-18.07592\pi}{180}\right)$$

$$= \mathbf{24.611046}$$

$$e'_{x2} = e''_x \times \cos\left(\frac{\theta\pi}{180}\right) - e''_{z2} \times \sin\left(\frac{\theta\pi}{180}\right)$$

$$= 26.845941 \times \cos\left(\frac{-18.07592\pi}{180}\right) - (-15.629222) \times \sin\left(\frac{-18.07592\pi}{180}\right)$$

$$= \mathbf{20.671605}$$

$$e'_{y1} = e''_{y1}$$

$$= \mathbf{-15.592839}$$

$$e'_{y2} = e''_{y2}$$

$$= \mathbf{-2.7321666}$$

$$e'_{z1} = e''_x \times \sin\left(\frac{\theta\pi}{180}\right) + e''_{z1} \times \cos\left(\frac{\theta\pi}{180}\right)$$

$$= 26.845941 \times \sin\left(\frac{-18.07592\pi}{180}\right) + (-2.9326908) \times \cos\left(\frac{-18.07592\pi}{180}\right)$$

$$= \mathbf{-11.117627}$$

$$e'_{z2} = e''_x \times \sin\left(\frac{\theta\pi}{180}\right) + e''_{z2} \times \cos\left(\frac{\theta\pi}{180}\right)$$

$$= 26.845941 \times \sin\left(\frac{-18.07592\pi}{180}\right) + (-15.629222) \times \cos\left(\frac{-18.07592\pi}{180}\right)$$

$$= \mathbf{-23.187537}$$

Results

Our two possible e points:

$$e'_1 = (24.611046, -15.592839, -11.117627)$$

$$e'_2 = (20.671605, -2.7321666, -23.187537)$$

5.5.3 Z-Axis Rotation

$$e_{x1} = e'_{x1} \times \cos\left(\frac{\phi\pi}{180}\right) - e'_{y1} \times \sin\left(\frac{\phi\pi}{180}\right)$$

$$= 24.611046 \times \cos\left(\frac{70.81\pi}{180}\right) - (-15.592839) \times \sin\left(\frac{70.81\pi}{180}\right)$$

$$= \mathbf{22.816099}$$

$$e_{x2} = e'_{x2} \times \cos\left(\frac{\phi\pi}{180}\right) - e'_{y2} \times \sin\left(\frac{\phi\pi}{180}\right)$$

$$= 20.671605 \times \cos\left(\frac{70.81\pi}{180}\right) - (-2.7321666) \times \sin\left(\frac{70.81\pi}{180}\right)$$

$$= \mathbf{9.3751445}$$

$$e_{y1} = e'_{x1} \times \sin\left(\frac{\phi\pi}{180}\right) + e'_{y1} \times \cos\left(\frac{\phi\pi}{180}\right)$$

$$= 24.611046 \times \sin\left(\frac{70.81\pi}{180}\right) + (-15.592839) \times \cos\left(\frac{70.81\pi}{180}\right)$$

$$= \mathbf{18.118108}$$

$$e_{y2} = e'_{x2} \times \sin\left(\frac{\phi\pi}{180}\right) + e'_{y2} \times \cos\left(\frac{\phi\pi}{180}\right)$$

$$= 20.671605 \times \sin\left(\frac{70.81\pi}{180}\right) + (-2.7321666) \times \cos\left(\frac{70.81\pi}{180}\right)$$

$$= \mathbf{18.624893}$$

$$e_{z1} = e_z z1'$$

$$= \mathbf{-11.117627}$$

$$e_{z2} = e_z z2'$$

$$= \mathbf{-23.187537}$$

Results

Our two possible e points:

$$e_1 = (22.816099, 18.118108, -11.117627)$$

$$e_2 = (9.3751445, 18.624893, -23.187537)$$

5.5.4 Position shifting

$$e_x = e_x + (-28.75)$$

$$e_y = e_y + 25$$

$$e_z = e_z + 10.4375$$

Results

Our two possible e points:

$$e_1 = (-5.934, 43.118, -0.680)$$

$$e_2 = (-19.375, 43.625, -12.750)$$

5.6 Coordinate discrimination

Finally we can use distance de to discriminate between the points and get to the correct one.

$$\text{distance } de_1 = \sqrt{((-5.934) - (-34.093750))^2 + (43.118 - 26.218750)^2 + ((-0.680) - (-5.531250))^2}$$

$$= \mathbf{33.197753}$$

$$\text{distance } de_2 = \sqrt{((-19.375) - (-34.093750))^2 + (43.625 - 26.218750)^2 + ((-12.750) - (-5.531250))^2}$$

$$= \mathbf{23.910866}$$

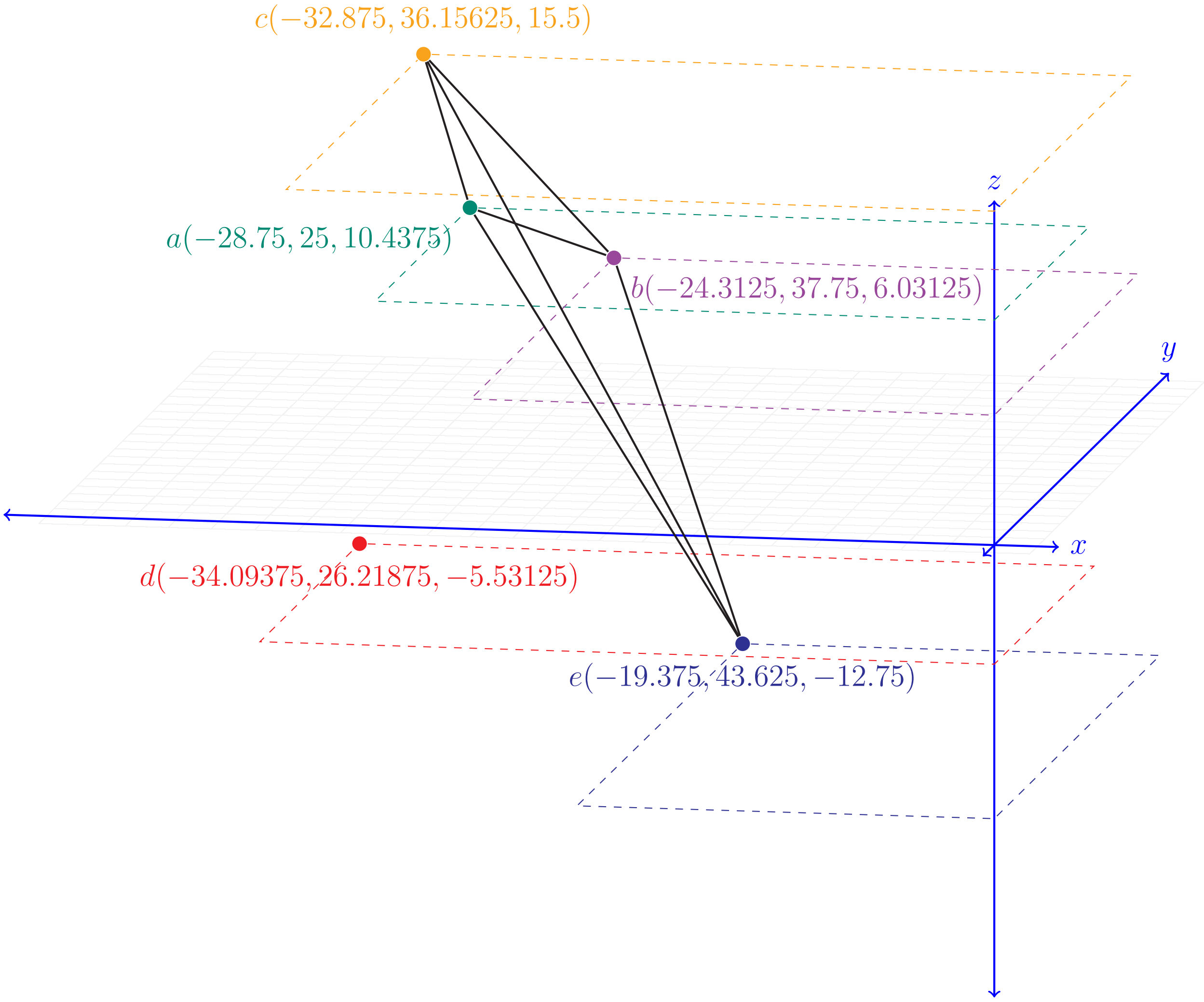
de_2 matches the known de distance of 23.9109 l.y..

5.7 Conclusion

Results

e_2 is our correct coordinate. Time to go exploring...

$$e = (-19.375, 43.625, -12.750)$$


$c(-32.875, 36.15625, 15.5)$
z
$a(-28.75, 25, 10.4375)$
$b(-24.3125, 37.75, 6.03125)$
y
x
$d(-34.09375, 26.21875, -5.53125)$
$e(-19.375, 43.625, -12.75)$